

Quantum Anomalous Hall Effect in $\text{Hg}_{1-y}\text{Mn}_y\text{Te}$ Quantum Wells

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The quantum Hall effect is usually observed when the two-dimensional electron gas is subjected to an external magnetic field, so that their quantum states form Landau levels. In this work we predict that a new phenomenon, the quantum anomalous Hall effect, can be realized in $\text{Hg}_{1-y}\text{Mn}_y\text{Te}$ quantum wells, without the external magnetic field and the associated Landau levels. This effect arises purely from the spin polarization of the Mn atoms, and the quantized Hall conductance is predicted for a range of quantum well thickness and the concentration of the Mn atoms. This effect enables dissipationless charge current in spintronics devices.

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When the two-dimensional electron gas (2DEG) is subjected to a high magnetic field, electronic states form Landau levels. In the regime where the temperature is low compared to the spacing between the Landau levels, quantized Hall conductance can be observed. In the quantum Hall (QH) regime, the electric current flows unidirectionally along the edge of the sample. Since back-scattering is absent, the edge current flows without any dissipation. The breaking of the time reversal symmetry is a necessary condition for the Hall effect. However, an external magnetic field is not required. Soon after the observation of the Hall effect, Edwin Hall also observed the anomalous Hall effect[1], where an additional Hall resistance arises from the spin-orbit interaction between the electric current and the magnetic moments. In the extreme case, an anomalous Hall effect can occur without the external magnetic field as long as the system breaks time-reversal symmetry spontaneously. Given the experimental observation of the QH effect, it is natural to ask whether the anomalous Hall effect can also be quantized without the external magnetic field and the associated Landau levels. Such a question is not only of great academic interests, but also has important practical implications. Realizing dissipationless charge current through the quantum anomalous Hall (QAH) effect without an external magnetic field could enable a new generation of quantum electronic devices.

Some years ago, Haldane[2] constructed a theoretical toy model to show that the QH effect is in principle possible without Landau levels. The model describes the hopping of spinless fermions on a honeycomb lattice, where the next-nearest neighbor hopping has complex amplitudes. Even though the complex hopping amplitude breaks the time reversal symmetry, it does not break the lattice translation symmetry since the net magnetic flux vanishes inside a unit cell. Haldane's model was extended to include localization physics in Ref.[3]. Unfortunately, this model is mostly academic, and can not be realized in the recently discovered graphene system. Later, the work on intrinsic anomalous Hall effect of ferromagnetic semiconductor in the metallic regime[4, 5, 6] shows the relation between the Berry's phase and Hall conduc-

tance. Qi, Wu and Zhang[7] constructed a tight-binding model of electrons spin-orbit coupled to polarized magnetic moments, and showed that the Hall conductance can be quantized in appropriate parameter regimes. This model breaks the time reversal symmetry due to magnetic moments rather than Landau levels, and provides another example of the QAH effect. More recently, a closely related topological phenomenon known as quantum spin Hall (QSH) effect has been theoretically predicted and experimentally observed in HgTe quantum wells[8, 9]. In this work, we show that when the HgTe quantum wells are doped with the magnetic Mn atoms, the QAH effect can be realized within an experimentally accessible parameter regime. We propose an experiment to demonstrate that the quantized Hall conductance indeed arises from the magnetic moments rather than Landau levels.

As a starting point, we first briefly review the physics of QSH effect in HgTe quantum wells. HgTe has an inverted band structure, where the p-type Γ_8 band has higher energy compared to the s-type Γ_6 band at the Γ point. For HgTe/CdTe quantum wells, there exists a topological quantum phase transition across some critical well thickness d_c where the band structure changes from the normal to the inverted character. The novel QSH effect occurs in the inverted regime $d > d_c$ [8]. In order to describe the physics near d_c , an effective four-band model is introduced as

$$H_0(k) = \begin{pmatrix} h_+(k) & 0 \\ 0 & h_-(k) \end{pmatrix}, \quad (1)$$

where

$$h_+(k) = \begin{pmatrix} \epsilon_k + \mathcal{M}(k) & Ak_+ \\ Ak_- & \epsilon_k - \mathcal{M}(k) \end{pmatrix}. \quad (2)$$

and $h_-(k) = h_+^*(-k)$ as is required by time-reversal symmetry. ϵ_k and $\mathcal{M}(k)$ can be in general expanded as

$$\epsilon_k = C_0 + C_2k^2, \quad \mathcal{M}(k) = M_0 + M_2k^2. \quad (3)$$

This effective model is expressed in the subspace containing the states $|E1, \pm\rangle$ and $|H1, \pm\rangle$, where $|E1, \pm\rangle$ is a superposition of $|\Gamma_6, \pm\frac{1}{2}\rangle$ and $|\Gamma_8, \pm\frac{1}{2}\rangle$ states, while $|H1, \pm\rangle$

is formed by $|\Gamma_8, \pm \frac{3}{2}\rangle$ states. Here \pm denote the two spin states which are degenerate due to the Kramers theorem when the time reversal symmetry is preserved. The diagonal block $h_{\pm}(k)$ describes a Dirac model in 2+1 dimensions, which at half filling carries a Hall conductance of $\pm e^2/h$, respectively[2, 7, 8]. Thus the net Hall conductance of the inverted quantum well system vanishes, while the spin Hall conductance, defined as the difference between the two blocks, is still non-zero. Therefore the QSH effect can be viewed as two copies of the QAH effects, with the opposite quanta of Hall conductances.

When the time reversal symmetry is broken, the two spin blocks are no longer related, and their charge Hall conductances no longer cancel exactly. *The key idea of this work is to identify the parameter space where one spin block is in the normal regime, while the other spin block is in the inverted regime.* The normal regime gives a topologically trivial insulator with vanishing Hall conductance, while the inverted regimes gives a topologically non-trivial insulator with one quantum unit of the Hall conductance; therefore, the whole system becomes a QAH state. Now we return to the four band effective Hamiltonian (1) and address what kind of term can induce the QAH effect. To describe the spin splitting induced by the magnetization, a phenomenological term is introduced as

$$H_s = \begin{pmatrix} G_E & 0 & 0 & 0 \\ 0 & G_H & 0 & 0 \\ 0 & 0 & -G_E & 0 \\ 0 & 0 & 0 & -G_H \end{pmatrix}. \quad (4)$$

where the spin splitting is $2G_E$ for the $|E1, \pm\rangle$ band and $2G_H$ for the $|H1, \pm\rangle$ band. Then the energy gap is given by $E_g^+ = 2M_0 + G_E - G_H$ for the up spin block while $E_g^- = 2M_0 - G_E + G_H$ for the down spin block. In order to obtain the QAH effect, we require that (1) the state with one kind of spin is in the inverted regime while the other goes into the normal regime, namely $E_g^+ E_g^- < 0$; (2) the entire system is still in the insulating phase with a full bulk gap, which requires that the states $|E1, +\rangle$ ($|E1, -\rangle$) and $|H1, -\rangle$ ($|H1, +\rangle$) do not cross each other, leading to the condition $(2M_0 + G_E + G_H)(2M_0 - G_E - G_H) > 0$. Combining the two conditions discussed above, we arrive at the condition $G_E G_H < 0$, which requires that the spin splittings for $|E1, \pm\rangle$ and $|H1, \pm\rangle$ must have the opposite sign. This condition is illustrated in Fig. 1(a). We can also understand the physics from the edge state picture (Fig 1 (b)). On the boundary of a QSH insulator there are counter-propagating edge states carrying opposite spin. When the spin splitting term increases, the spin down edge states penetrate much deeper into the bulk due to the decreasing gap, and eventually disappear, leaving only the spin up state bound more strongly to the edge. Thus the system has only spin up edge states and transforms from the QSH state to the QAH state, as illustrated in Fig 1 (b).

We have therefore identified a key condition $G_E G_H < 0$, for the QAH effect. We may ask whether or not

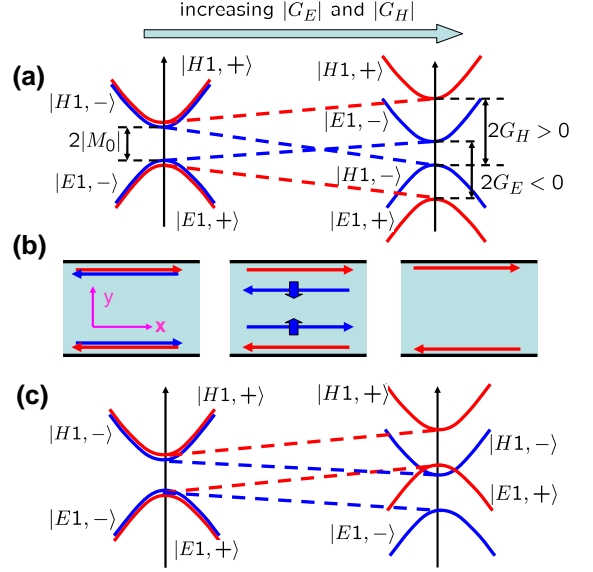


FIG. 1: (Color online) Evolution of band structure and edge states upon increasing the spin splitting. For (a) $G_E < 0$ and $G_H > 0$, the spin down states $|E1, -\rangle$ and $|H1, -\rangle$ in the same block of Hamiltonian (1) touch each other and then enters the normal regime. But for (c) $G_E > 0$ and $G_H > 0$, gap closing occurs between $|E1, +\rangle$ and $|H1, -\rangle$ bands, which belongs to different blocks of Hamiltonian, and thus will cross each other without opening a gap. In (b), we show the behavior of the edge states during the level crossing in the case of (a).

it is true in the realistic material. Fortunately, in the $\text{Cd}_x\text{Mn}_y\text{Hg}_{1-x-y}\text{Te} / \text{Mn}_y\text{Hg}_{1-y}\text{Te}$ quantum well, the sp-d exchange coupling indeed gives the opposite signs for G_E and G_H , a fact which is well established in the literature[10, 11]. From a standard perturbative treatment of the eight band Kane model[8, 12], we find for a spin polarization perpendicular to the quantum well plane, the coefficients G_E , G_H in the four-band effective model (4) can be expressed as

$$G_E = -(3AF_1 + BF_4), \quad G_H = -3B, \quad (5)$$

in which A, B are given by[11, 13]

$$A = \frac{1}{6}N_0\alpha y\langle S \rangle, \quad B = \frac{1}{6}N_0\beta y\langle S \rangle, \quad (6)$$

and F_1, F_4 are the amplitudes of $|\Gamma_6, \pm \frac{1}{2}\rangle$ and $|\Gamma_8, \pm \frac{1}{2}\rangle$ components in the state $|E1, \pm\rangle$, respectively. Here N_0 is the number of unit cells per unit volume and $\langle S \rangle$ is the spin polarization of Mn out of quantum well plane. α and β describe the sp-d exchange coupling strength for s-band and p-band electron respectively, where the signs and magnitudes are crucial for the relative sign of G_E and G_H . For $\text{Hg}_{1-y}\text{Mn}_y\text{Te}$, these parameters are determined to be $N_0\alpha = 0.4\text{eV}$ and $N_0\beta = -0.6\text{eV}$ [11, 13], leading to the opposite signs for G_E and G_H . Combined with the previous analysis about the Hamiltonian (4), we conclude that QAH effect can occur in the

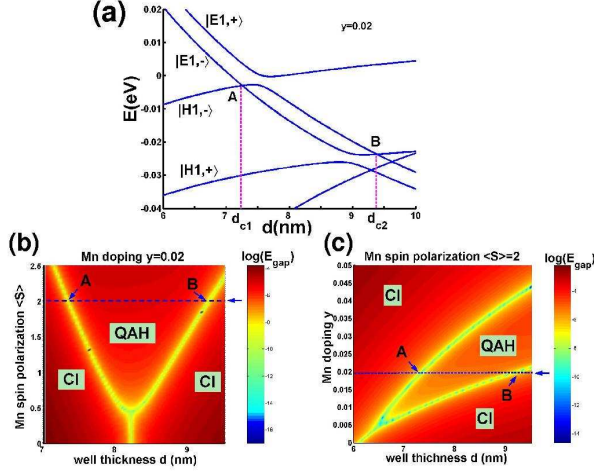


FIG. 2: (a) The energy levels for $|E1, \pm\rangle$ and $|H1, \pm\rangle$ are plotted as a function of the quantum well thickness. Two crossing points (A and B) are labelled in the figure. The energy gap ($\log(E_{gap})$ used here) is plotted as a function of the well thickness d versus the Mn magnetic moment $\langle S \rangle$ in (b), versus the Mn doping concentration y in (c). Dashed blue line in (b) or (c) refers to the line along which (a) is plotted. The points 'A' and 'B' correspond to the two Dirac type crossing points. Two different phases, conventional insulator (CI) with $\sigma_H = 0$ and QAH state with $\sigma_H = -e^2/h$, are separated by the gap closing line in the figures.

$Cd_xMn_yHg_{1-x-y}Te/Mn_yHg_{1-y}Te$ quantum well, as long as the Mn magnetization $\langle S \rangle$ is large enough and perpendicular to the quantum well plane.

Though the analysis above already gives a clear explanation to the physical mechanism of QAH effect, to obtain more quantitative predictions we perform a more realistic calculation of the electronic structure based on the eight band Kane model[12, 13]. Our Hamiltonian takes into account the exchange term and bulk inversion asymmetry (BIA) terms[12], in addition to the terms included in the original Kane model. In Fig. 2 (a), the energy spectrum of the lowest subbands $|E1, \pm\rangle$ and $|H1, \pm\rangle$ at the Γ point is plotted as a function of quantum well thickness d . At a critical thickness $d_{c1} = 7.25$ nm, a level crossing 'A' occurs between the $|E1, -\rangle$ and the $|H1, -\rangle$ bands. This is a Dirac-type level crossing, which corresponds to a topological quantum phase transition, across which the Hall conductance jumps by $-e^2/h$. Since the system at $d < d_{c1}$ always remains gapped when the Mn magnetization is adiabatically turned on, we know $\sigma_H = 0$ for $d < d_{c1}$ and $\sigma_H = -e^2/h$ for $d > d_{c1}$. The same analysis applies to the other critical thickness $d_{c2} = 9.4$ nm ('B' in Fig 2 (a)), where the level crossing occurs between $|E1, +\rangle$ and $|H1, +\rangle$ bands and the Hall conductance returns to 0. Therefore, for parameters $\langle S \rangle = 2$ and $y = 0.02$, QAH effect appears in a quantum well thickness range $d_{c1} < d < d_{c2}$. The same calculation can be carried for different spin polarizations $\langle S \rangle$ or y to

determine the whole phase diagram with three tunable parameters: quantum well thickness d , spin polarization $\langle S \rangle$ and Mn fraction y . In Fig. 2 (b) and (c), the gap sizes in the d - $\langle S \rangle$ plane and the d - y plane are plotted, respectively. The line with bright color shows the phase boundary with vanishing gap. At zero $\langle S \rangle$ or zero y limit, the time-reversal invariance is recovered and the critical line terminates at the transition point between trivial and QSH insulators[8]. An important feature of the phase diagram is that a minimal $\langle S \rangle$ (for fixed y) or minimal y (for fixed $\langle S \rangle$) is required to obtain QAH phase. For example, for $y = 0.02$ we need $\langle S \rangle > 0.5$ for the QAH phase. This is a consequence of bulk inversion asymmetry (BIA).

We now discuss the experimental observation of the QAH effect in the $Hg_{1-y}Mn_yTe$ system. The main difficulty lies in the fact that $Hg_{1-y}Mn_yTe$ is paramagnetic rather than ferromagnetic. Thus the spin polarization is determined by the Curie-Weiss formula:

$$\langle S \rangle = -S_0 B_{5/2} \left(\frac{5g_{Mn}\mu_B B}{2k_B(T + T_0)} \right) \quad (7)$$

where $S_0 = 5/2$ and $B_{5/2}$ is the Brillouin function[12, 13], and $T_0 > 0$ stands for a weak antiferromagnetic coupling between Mn spins. Hence, we need to apply a small magnetic field to polarize the Mn moments and generate the QAH effect. In fact, experiments carried out at the University of Würzburg has already shown quantized Hall conductance for magnetic fields as small as 1T[14]. However, since even such a small magnetic field still has an orbital effect, this experiment by itself cannot prove the existence of the QAH, which arises purely from the magnetic moments of Mn . To solve this problem, we propose two different ways to polarize Mn spins without an orbital magnetic field. The first approach is to apply a low-frequency polarized infrared light to provide the angular momentum required for aligning the Mn moments. Such a photo-induced Mn magnetization has been observed experimentally both in $Hg_{1-x}Mn_xTe$ and $Cd_{1-x}Mn_xTe$ [15, 16]. Though this proposal has the advantage of realizing this effect in a steady state, another approach, the time-resolved Hall measurement, can provide a more dramatic demonstration of the QAH effect. Thus in the following we will focus on the time-resolved Hall measurement proposal. Firstly, a static magnetic field is applied, which acts on the itinerant electrons through three terms: the orbital effect H_{orb} , the Zeeman term H_Z and the exchange term H_{ex} . In H_{ex} , the Mn spin polarization is determined by Eq. (7). Secondly, at time t_0 the magnetic field is switched off within a time scale τ_B . As a result, the Mn spin polarization will decay to zero in a spin-relaxation time τ_s . However, if $\tau_B \ll \tau_s$, there is a time window $t_0 + \tau_B < t \ll t_0 + \tau_s$ when the orbital and Zeeman effect of the magnetic field already disappeared, but the Mn spin polarization remains similar to the value before magnetic field is removed. Consequently, within this time range there is no conventional QH based on Landau levels, so that the pure QAH effect

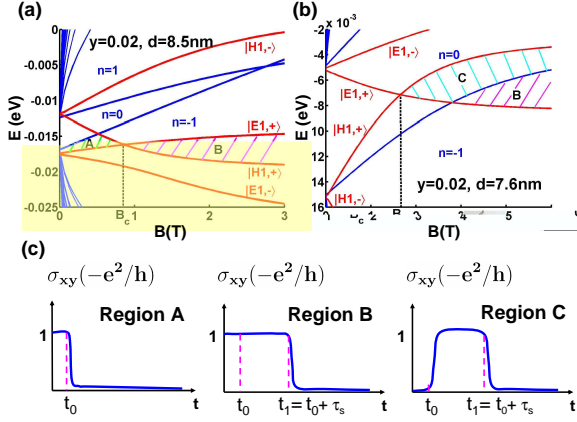


FIG. 3: Landau level spectra are plotted as a function of magnetic field (blue line) for two different thicknesses (a) $d = 8.5\text{nm}$ and (b) $d = 7.6\text{nm}$, taking into account exchange term, Zeeman splitting and orbital effect. The corresponding red lines show the band edge of the sub-bands $|H1, \pm\rangle$ and $|E1, \pm\rangle$ when the magnetic field has been turned off but the Mn spin polarization remains. Without an external magnetic field, the system is in a QSH insulator phase for (a), and trivial insulator phase for (b). Figure (c) shows schematically the predicted Hall conductances for the three different regions A, B and C defined by shadows in figure (a) and (b).

can be observed, if the Mn spin polarization stays in the correct range.

To illustrate this proposal more clearly, we compare the band structure with and without Landau levels. In Fig 4 (a) and (b), the energy spectra is plotted as a function of the magnetic field for different thicknesses. The blue lines take into account the orbital magnetic field term, the exchange and the Zeeman terms, corresponding to the spectra at time $t < \tau_B$, (setting $t_0 = 0$). In contrast, the red lines are the conduction and valence band edges of the system at time $\tau_B < t \ll \tau_s$, after the orbital and Zeeman effects of the magnetic field are switched off but the Mn spin polarization remains the same. Since the energy spectrum is dispersive when the orbital effect of magnetic field is absent, the system is insulating only when the fermi energy lies between the two middle red lines, *i.e.*, between $|E1, -\rangle$ and $|H1, -\rangle$ for Fig. 4 (a), or between $|E1, +\rangle$ and $|H1, +\rangle$ for Fig. 4 (b). Depending on the initial values of fermi energy E_F and magnetic field B , three different phenomena can be observed in the proposed experiment. (i) When (E_F, B) is in Region (A) of Fig. 4 (a), the system has Hall conductance $\sigma_H = -e^2/h$ in the static magnetic field, and enters a trivial insulator phase after magnetic field is switched off. Consequently, the Hall conductance will drop to zero once the magnetic field is removed. (ii) When (E_F, B)

is in Region (B) of Fig. 4 (a) and (b), the system has Hall conductance $\sigma_H = -e^2/h$ for static field and enters a QAH phase with the same Hall conductance after the magnetic field is switched off. Thus the Hall conductance will remain on the $-e^2/h$ plateau for a time $\sim \tau_s$ after magnetic field is turned off. (iii) When (E_F, B) is in Region (C) of Fig. 4 (b), the system has vanishing Hall conductance in static field, but enters a QAH phase with $\sigma_H = -e^2/h$ after the magnetic field is switched off. Consequently, we will observe the dramatic appearance of a “pulse” of the Hall conductance in a time-scale $\sim \tau_s$ even though the system is in the $\sigma_H = 0$ Hall plateau under a static field! Observation of this phenomenon gives the conclusive demonstration of the QAH effect. Due to the topological distinction between $\sigma_H = 0$ and $\sigma_H = -e^2/h$ states, the transition between them is always sharp at low temperature, even though the Mn magnetization changes continuously. Consequently, the time-dependence of σ_{xx} and σ_{xy} in this proposal should show the same critical behavior as the usual “plateau transition” in the QH effect.

Finally, we estimate the experimental conditions required to realize this proposal. First of all, both of the switch-off time τ_B and the time resolution δt [17] of the Hall measurement should be shorter than Mn spin relaxation time τ_s , which is of the order $10 - 100\mu\text{s}$ [18, 19]. Further, the magnetic field should be turned off slowly enough for electrons to stay in the instantaneous ground state during the switch-off operation (adiabatic condition), which requires $\tau_B \gg \hbar/E_g$, where E_g is the energy gap and $\hbar/E_g \sim 10^{-12}\text{s}$ in the present system. Therefore, a magnetic field of several Tesla needs to be switched off in a time scale of $10^{-12}\text{s} \ll \tau_B \ll 10^{-4}\text{s}$, and the time-resolution of transport measurement should satisfy $\delta t \ll 10^{-4}\text{s}$.

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